

# Is the regime with shot noise suppression by a factor $1/3$ achievable in semiconductor devices with mesoscopic dimensions?

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We discuss the possibility of diffusive conduction and thus of suppression of shot noise by a factor  $1/3$  in mesoscopic semiconductor devices with two-dimensional and one-dimensional potential disorder, for which existing experimental results do not provide a conclusive result. On the basis of our numerical analysis, we conclude that it is quite difficult to achieve diffusive transport over a reasonably wide parameter range, unless the device dimensions are increased up to the macroscopic scale. In addition, in the case of one-dimensional disorder, some mechanism capable of mode-mixing has to be present in order to reach or even approach the diffusive regime.

## I. INTRODUCTION

In nanoscale devices shot noise (i.e. the noise due to the discreteness of the electron charge) is often suppressed with respect to the value of power spectral density given by the Schottky formula  $S_I = 2e|I|$  (where  $e$  is the elementary charge and  $I$  is the average current), which would be expected in the absence of correlations between the carriers<sup>1</sup>. The ratio that quantifies this suppression, due to the correlations introduced by Coulomb interactions or (as in the cases discussed in this article) by Fermi exclusion, is called Fano factor.

Particularly interesting is the case of diffusive conductors. It is well known<sup>2</sup> that, if the length  $L$  of the device is much less than the elastic mean free path  $l$ , the transport regime is ballistic and shot noise is strongly suppressed, while, if  $L$  is greater than  $Nl$  (where  $N$  is the number of propagating modes) and the conductor is assumed to be phase coherent (as we will do in this work), transport is characterized by strong localization, with the resistance increasing exponentially with the length of the device and the Fano factor approaching unity. Instead, in the intermediate regime in which the values of  $l$  (determined by the disorder inside the device),  $L$  and  $N$  satisfy the inequalities  $l \ll L \ll Nl$ , transport is diffusive. In this regime the resistance increases linearly with length and, being the distribution of the transmission eigenvalues  $T$  bimodal and proportional to  $1/(T\sqrt{1-T})$ , shot noise is suppressed by a factor  $1/3$ , due to the presence of a few open quantum channels with nearly unit transmission probability<sup>3,4</sup>. This result, in the case of devices with more than one-dimensional disorder, has been theoretically obtained using quantum-mechanical<sup>5,6</sup> or semiclassical<sup>7</sup> approaches, both of which include the effect of Fermi exclusion. The same result has been obtained in the case of one-dimensional disorder (i.e. a series of unevenly spaced tunnel barriers) using a semiclassical model<sup>8</sup>.

In this paper we will try to answer the question whether in semiconductor devices with mesoscopic dimensions it is possible to reach the diffusive transport regime and thus the corresponding  $1/3$  suppression of shot noise over a wide parameter range.

From the experimental point of view, Henny *et al.*<sup>9</sup> measured the shot noise suppression factor in a thin metallic wire, finding the theoretically predicted  $1/3$  reduction by asymptotically widening the contact reservoirs and thus reducing the reservoir heating which otherwise would increase the measured noise.

Instead, in the case of semiconductor devices, the existing experimental data are not conclusive. Liefink *et al.*<sup>10</sup> reported, for a wire obtained confining the two-dimensional electron gas of a GaAs/AlGaAs heterostructure, shot noise reduction factors varying between 0.2 and 0.45, depending on the width of the wire. Instead Song *et al.*<sup>11</sup> have measured the Fano factor for a semiconductor device with one-dimensional disorder, in particular for a superlattice in which carriers have been injected into the conduction band through optical excitations from the valence band. Also in this case, for low applied fields, the measured Fano factor has values that strongly differ from  $1/3$ , varying from one superlattice to another and in particular typically increasing with barrier width (i.e. decreasing barrier transparency).

Prompted by these results, we have performed several numerical investigations of the noise behavior of semiconductor structures characterized by 2-dimensional (2D) and one-dimensional (1D) disorder, using different models, with varying degree of approximation. In the case of 2D disorder, our conclusion is that in a semiconductor device with mesoscopic dimensions it is very difficult to obtain a diffusive behavior over a relatively large range of scatterer concentration or disorder strength: in order to reach such a result in a definite way, we should increase the number of propagating modes up to several thousands and thus consider a macroscopic device. On the other hand, in the case of 1D disorder we have found that the absence of coupling among the modes propagating in the structure makes it inherently impossible to obtain a diffusive regime, and thus a  $1/3$  Fano factor, unless some additional contribution (such as a magnetic field) is introduced to create mode mixing.

## II. TWO-DIMENSIONAL DISORDER

The structure we consider is a quantum wire obtained laterally confining, by means of negatively biased gates located on the surface, the two-dimensional electron gas (2DEG) of a GaAs/AlGaAs heterostructure. The ionized donors located inside the n-doped AlGaAs layer (together with other charged impurities present inside the heterostructure) determine potential fluctuations at the 2DEG level.

In a previous paper by our group<sup>12</sup> a self-consistent calculation of the average potential, combined with a semi-analytical formula for the effect of ionized donors, was used to fit the conductance measurement on a fabricated quantum wire and to numerically predict its noise behavior. In that case it was found that the Fano factor did not stabilize at  $1/3$ , but it rather crossed it for a single value of the gate bias voltage.

In order to gain a better understanding of the problem, we have now studied the noise behavior of the structure for a larger range of parameters, for some choices of a model potential. In detail, we have considered a  $4.9 \mu\text{m}$  long and  $8.4 \mu\text{m}$  wide conductor with a hard-wall lateral confinement (since in our previous investigations the detailed shape of the confinement potential did not appear to play a significant role on the noise behavior). We have considered that all of the dopants are located at a distance  $D = 40 \text{ nm}$  from the 2DEG and, for each considered dopant concentration

(with a uniform random distribution), we have initially evaluated the effect, at the level of the 2DEG, by summing up each individual contribution. The contribution of a single dopant has been evaluated with the semi-analytical expression given by Stern and Howard<sup>13</sup>, according to which a point charge  $Ze$  located at a distance  $D$  from the 2DEG generates on the 2DEG, at a distance  $r$  from its orthogonal projection onto the 2DEG plane, a screened potential equal to

$$\phi(r) = \frac{Ze}{4\pi\epsilon_0\epsilon_r} \int_0^\infty \frac{k}{k+s} J_0(kr) e^{-kD} dk \quad (1)$$

where  $\epsilon_0$  is the vacuum permittivity,  $\epsilon_r$  is the relative permittivity of the semiconductor,  $J_0$  is the Bessel function of order 0 and the screening constant is assumed equal to  $s = 2n_\nu(m^*e^2)/(4\pi\epsilon_0\epsilon_r\hbar^2)$  (with  $n_\nu = 1$  the considered subband degeneracy). In order to have a neutral system with a potential landscape symmetric around zero, which simplifies the investigation and comparison of a large number of different cases, we have considered an artificial situation with an equal number of positive and negative charges. Then, different disorder strengths for each dopant concentration have been obtained simply by multiplying the thus obtained potential profile by a scale factor  $K$ , thereby reducing the computational effort.

As an example, in Fig. 1 we show a map of the potential obtained at the 2DEG level for a concentration  $N_D = 1.1 \times 10^{14} \text{ m}^{-2}$  of impurities located at a distance  $D = 40 \text{ nm}$  from the 2DEG, multiplied by a disorder strength scale factor  $K = 39$ .

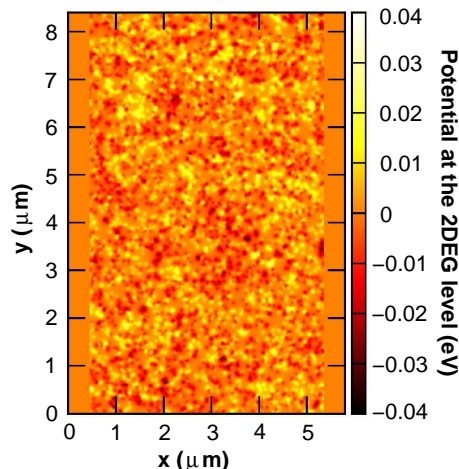


FIG. 1. Map of the potential obtained at the 2DEG level for a concentration  $N_D = 1.1 \times 10^{14} \text{ m}^{-2}$  of impurities located at a distance  $D = 40 \text{ nm}$  from the 2DEG, multiplied by a scale factor  $K = 39$ .

Once the potential at the 2DEG level has been obtained, the transmission matrix of the device has been evaluated using the recursive Green's function technique, with a representation over the transverse modes in the confined direction and in real space in the transport direction<sup>15,16</sup>. From the transmission matrix  $t$ , the conductance  $G$ , the shot noise power spectral density  $S_I$  and the Fano factor  $\eta$  have been obtained using the relations<sup>14</sup>:

$$G = \frac{2e^2}{h} \sum_i w_i, \quad S_I = 4 \frac{e^3}{h} |V| \sum_i w_i(1 - w_i), \quad (2)$$

$$\eta = \frac{S_I}{2e|I|} = \frac{\sum_i w_i(1 - w_i)}{\sum_i w_i}, \quad (3)$$

where  $V$  is the mean value of the externally applied voltage,  $h$  is Planck's constant and the  $w_i$ 's are the eigenvalues of the matrix  $t^\dagger t$ .

In these calculations we have separately averaged the conductance and noise results (and thus the numerator and denominator of Eq. (3)) over 41 energy values uniformly spaced in an energy range of  $80 \mu\text{eV}$  around  $E_F = 9 \text{ meV}$ .

In Fig. 2 we report the Fano factor that we have obtained for 7 values of the dopant concentration  $N_D$ , as a function of the disorder strength scale factor  $K$ . We see that for elevated concentrations the interval of disorder strength within which the curves approach the value  $1/3$  is very narrow, while it gets wider for low concentrations. However, in this

latter case the diffusive behavior is obtained only for very large scale factors  $K$ . If we focus our attention on the potential deriving, at the 2DEG level, from each single charged impurity and in particular on the portion that more affects transport, i.e. that above the Fermi energy, we see that for these values of  $K$  its spatial extent is of the order of hundreds of nanometers. Since this extension is unrealistic for semiconductor devices (while it could be reasonable for metallic conductors, characterized by the presence of large grains), we conclude that in semiconductor nano-devices it is quite unlikely to obtain a  $1/3$  shot noise suppression factor within a reasonably large parameter range. Finally, for the lowest concentration ( $N_D = 1.1 \times 10^{12} \text{ m}^{-2}$ ) the Fano factor remains well below the  $1/3$  value and thus we have a substantially ballistic regime for all the considered disorder strengths.

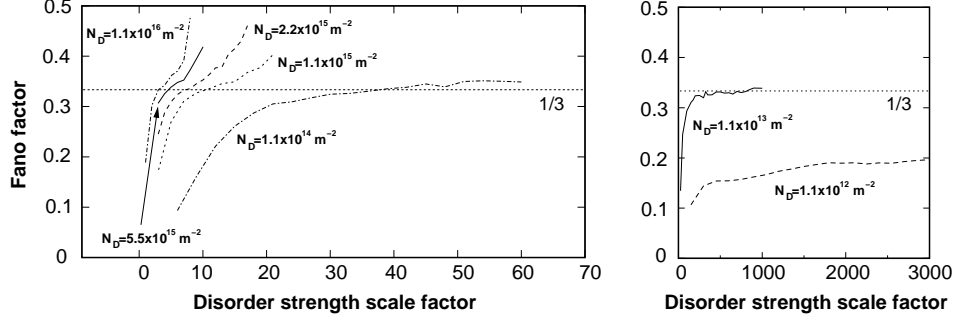


FIG. 2. Fano factor as a function of the disorder scale factor  $K$  for 7 values of the dopant concentration  $N_D$  and for  $E_F = 9 \text{ meV}$ .

In Fig. 3 we show the normalized conductance  $G/G_0$  (where  $G_0$  is the conductance quantum  $2e^2/h$ ) as a function of the disorder strength scale factor  $K$  for the same impurity concentrations  $N_D$ .

The relationship between the conductance  $G$ , the mean free path  $l$  and the device length  $L$  is approximately given by  $G/G_0 = Nl/(L+l)$ , with  $N$  the number of propagating modes<sup>2</sup>. In this case, being  $N = 336$ , the condition for diffusive transport  $l \ll L \ll Nl$  is satisfied by a factor of 10 for both inequalities if  $9.7 < G/G_0 < 30.5$ .

We again observe that the interval in which the condition for diffusive transport is satisfied is very narrow for elevated concentrations, while it widens for low concentrations, for which, however, large disorder strengths are needed. Finally, in the case of  $N_D = 1.1 \times 10^{12} \text{ m}^{-2}$ , the conductance never satisfies such a condition.

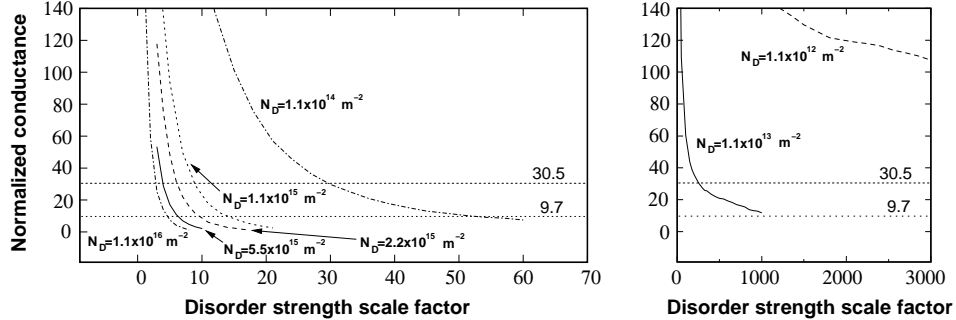


FIG. 3. Conductance (normalized with respect to the conductance quantum  $G_0$ ) as a function of the disorder scale factor  $K$  for 7 values of the dopant concentration  $N_D$  and for  $E_F = 9 \text{ meV}$ .

The comparison between Fig. 2 and Fig. 3 confirms that the Fano factor assumes values near  $1/3$  in the same parameter range in which the condition  $l \ll L \ll Nl$  is satisfied. This range can clearly be enlarged increasing the number  $N$  of propagating modes, but this can be obtained only considering wider conductors, with macroscopic dimensions, thus outside the mesoscopic range we are interested in.

In the case of metallic conductors, due to the higher number of propagating modes and to the possibility of extended scatterers, diffusive transport can actually be reached and experimentally measured.

Finally, in order to compare these conclusions with some previously obtained results<sup>17</sup>, we have performed some simulations using a more simplified model for the disordered potential. In detail, in Ref.<sup>17</sup> a discussion on the exact conditions needed to obtain the diffusive transport regime was presented, considering wires where the disordered potential was simulated with a random distribution of square obstacles.

In particular, we have repeated the calculation corresponding to the upper curve of Fig. 6 of Ref.<sup>17</sup>, obtaining the results reported in the upper panel of Fig. 4. The simulation has been performed considering a wire with a width  $W = 5 \mu\text{m}$  and a length  $L = 8 \mu\text{m}$ , containing 300 hard-wall square obstacles, with a 100 nm edge. In the figure we show the Fano factor as a function of the Fermi energy of the impinging electrons. We see that, as soon as the Fermi energy has reached the value corresponding to a number of propagating modes for which the condition  $l \ll L \ll Nl$  is satisfied, the Fano factor reaches the value  $1/3$ .

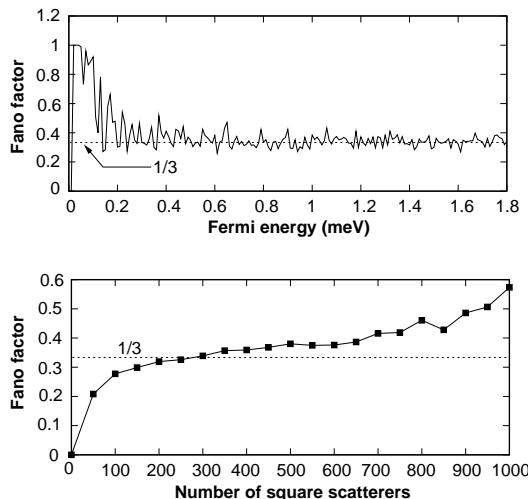


FIG. 4. Upper panel: Fano factor as a function of the Fermi energy of the impinging electrons, obtained in a  $5 \mu\text{m}$  wide and  $8 \mu\text{m}$  long wire, containing 300 hard-wall  $100 \text{ nm} \times 100 \text{ nm}$  obstacles. Lower panel: Fano factor as a function of the number of hard-wall square scatterers in the wire.

In order to verify whether the diffusive regime is preserved over a large range of scatterer concentrations, we have made some simulations varying the number of square scatterers inside the wire and computing the Fano factor around a value of the Fermi energy,  $E_F = 1.25 \text{ meV}$ , for which the diffusive regime has been reached in the case of 300 scatterers. In particular, we have separately averaged the conductance and noise results (and thus the numerator and denominator of Eq. (3)) over a set of 41 uniformly spaced energy values in a range of  $80 \mu\text{eV}$  around  $1.25 \text{ meV}$ . In the lower panel of Fig. 4 we show the behavior of the Fano factor as a function of the number of square scatterers inside the conductor. We see that also in this case the Fano factor does not settle around  $1/3$  for a large range of scatterer numbers, but just crosses the  $1/3$  value in correspondence of about 300 scatterers, which is indeed the situation for which the conditions for diffusive regime have been investigated in Ref.<sup>17</sup>. Thus, this was a particular case, not representative of the general behavior.

### III. ONE-DIMENSIONAL DISORDER

The case of strictly one-dimensional disorder<sup>18,19</sup>, i.e. of unevenly spaced tunnel barriers located in an otherwise purely ballistic device with any dimensionality, is a bit different from the cases of 2D or 3D disorder, even though, according to a semiclassical study of this structure<sup>8</sup>, also in this case the shot noise suppression factor should approach  $1/3$  as the number of cascaded barriers is let go to infinity.

A series of barriers can be defined, for example, in a heterostructure-based device defining a series of gates on the surface of the device. Such gates, when negatively biased, locally deplete the 2DEG, each generating a tunnel barrier for the electrons traveling in the device.

We consider, for simplicity, idealized rectangular barriers, although we have verified that the main results hold also for realistic barriers<sup>20</sup>. Since a wire with a series of such rectangular transversal barriers (each extending across the whole cross-section) can be seen as made up of a series of sections differing only for the value of their constant potential, the wave functions associated with the transverse modes are the same in all of the sections and the tunnel barriers do not introduce any mode-mixing. Therefore the transport calculation can be subdivided into many purely 1D problems. Using a scattering matrix approach, in which the scattering matrix of each barrier and of each interbarrier region has a well-known analytical form, we have found the transmission  $t_i$  for each mode  $i$  through the device. The conductance, shot noise power spectral density and Fano factor can then be obtained using Eqs. (2)-(3), in which,

due to the absence of mode-mixing, we have<sup>21</sup> that  $w_i = |t_i|^2$ .

In Fig. 5 we report, for a series of identical barriers, the Fano factor as a function of the number of the unevenly spaced barriers for 3 values of the barrier transparency  $\Gamma$ . In detail, we have considered a 8  $\mu\text{m}$  wide structure and we have averaged our conduction and noise results over 500 energy values uniformly distributed in a range of 40  $\mu\text{eV}$  around 9.03 meV. We have considered 0.425 nm thick barriers, with heights equal to 0.8, 0.25 and 0.07 eV. Defining the barrier transparency  $\Gamma$  as the squared modulus of the transmission through each barrier, averaged over all the propagating modes, for the 3 values of barrier height the transparency  $\Gamma$  is about equal to 0.1, 0.5 and 0.9, respectively. In order to obtain a general behavior, we have averaged the results over 50 different sets of interbarrier distances, which is equivalent to introducing dephasing with a simple phase randomization model preserving localization effects<sup>22</sup>. These averages show that, contrary to what was expected from a semiclassical analysis<sup>8</sup>, no common asymptotic 1/3 value for the Fano factor is reached increasing the number of barriers.

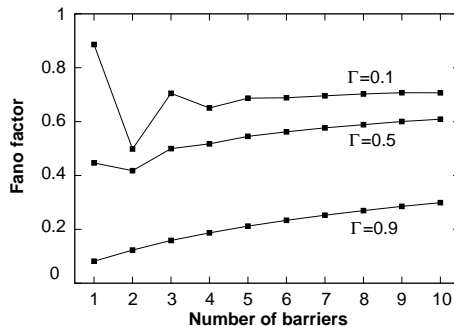


FIG. 5. Fano factor as a function of the number of barriers for 3 values of the barrier transparency  $\Gamma$ , averaged over 50 different sets of interbarrier distances.

The reason is that in the absence of mode-mixing the overall transport problem is just a collection of intrinsically one-dimensional problems; therefore the localization length  $L_l$  is equal to  $l$  and thus it is impossible to satisfy the condition for diffusive transport  $l \ll L \ll L_l$ . Since semiclassical descriptions do not take into account the effect of phase coherence on transport and thus do not include localization, this effect can not be predicted using semiclassical arguments, but results only from a quantum-mechanical analysis.

In Ref.<sup>20</sup> we have also shown that the presence of a realistic amount of edge-roughness in the depletion gates defining the barriers, introducing only a small degree of mode-mixing, does not alter significantly the described results.

On the other hand, the amount of 2D disorder that we should add to the device in order to create mode-mixing and reach the diffusive regime would be such that it would lead to diffusive transport even in the absence of the barriers, and therefore of the 1D disorder.

Therefore, in order to reach the diffusive regime while preserving the 1D nature of the disorder, we have to introduce a different source of mode-mixing, for example a magnetic field threading the device. We have shown elsewhere<sup>18</sup> that, especially in the case of barriers with high transparency, the presence of a magnetic field makes it possible to reach a 1/3 value for the Fano factor, by introducing mode-mixing and thus increasing the localization length with respect to the mean free path.

Here we show the results obtained for a 1  $\mu\text{m}$  wide conductor containing a series of 66 meV high and 1.56 nm thick barriers, with an average transparency at the considered Fermi energy (9.03 meV)  $\Gamma = 0.5$ . The transport calculation has been carried out using the recursive Green's function technique, and adopting, for the representation of the vector potential, a Landau gauge with nonzero component only along the transverse direction<sup>23</sup>. The conductance and noise results have been averaged over a set of 25 energy values uniformly distributed over a range of 40  $\mu\text{eV}$  around 9.03 meV. The final results have been averaged over 20 different sets of interbarrier distances. We see in Fig. 6 that, while in the absence of magnetic field we observe an exponential behavior of the resistance as a function of the number of the barriers (characteristic of the strong localization regime), applying an orthogonal magnetic field  $B = 0.1$  T the resistance behavior becomes approximately linear, i.e. we approach the diffusive regime.

However, in order to obtain a diffusive regime over a really wide range of parameters, the mode-mixing introduced by the magnetic field has to be combined with the presence of a large number of propagating modes, which requires, also in this case, macroscopic dimensions.

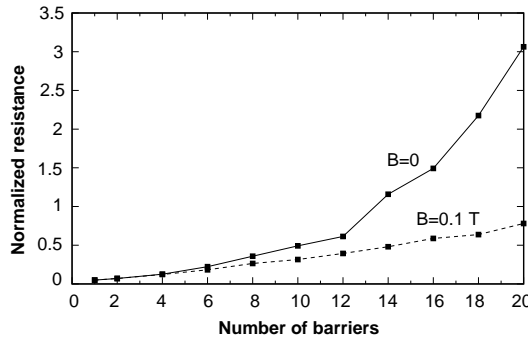


FIG. 6. Resistance (normalized with respect to the resistance quantum  $h/(2e^2)$ ) as a function of the number of cascaded tunnel barriers in a  $1\text{ }\mu\text{m}$  wide conductor with 66 meV high and 1.56 nm thick barriers, with  $E_F = 9.03\text{ meV}$ ; the exponential behavior has been obtained without the magnetic field, while the nearly linear one has been found applying an orthogonal magnetic field  $B = 0.1\text{ T}$ .

#### IV. CONCLUSION

We have investigated the possibility of diffusive transport, and thus of suppression of shot noise down to  $1/3$ , in mesoscopic semiconductor devices with 2D and 1D disorder.

From our numerical simulations, in which different representations for the potential disorder have been adopted, we have concluded that it should be very difficult and rather uncommon to obtain fully diffusive transport in mesoscopic semiconductor devices, due to the insufficient number of propagating modes. In addition, in the case of 1D disorder the absence of mode-mixing makes it theoretically impossible to reach the diffusive regime, unless a source of mode-mixing, such as a magnetic field, is present.

Our conclusions seem to be supported by existing experimental results, which in the case of mesoscopic semiconductor devices have not shown a clear  $1/3$  suppression of shot noise.

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